Eur. Phys. J. A **5**, 91–95 (1999)

THE EUROPEAN PHYSICAL JOURNAL A

The renormalon contribution to the current product $j_{\mu 5} j_S$

M. Meyer-Hermann[1], A. Schäfer[2]

[1] Institut für Theoretische Physik, TU Dresden, D-01062 Dresden, Germany
[2] Fakultät Physik, Universität Regensburg, D-93040 Regensburg, Germany

Received: 9 November 1998 / Revised version: 22 January 1999
Communicated by W. Weise

Abstract. The product of an axialvector and a scalar current and its relation to the chiral-odd distribution function h_1 is discussed in the framework of the renormalon approach. Using a bag-model calculation for h_1, we calculate its intrinsic uncertainty due to renormalon poles. The result is given as a function of Bjorken-x as well as for the first moments separately.

PACS. 12.38.Lg Other nonperturbative calculations – 14.20.Dh Protons and neutrons – 13.85.Qk Inclusive production with identified leptons, photons, or other nonhadronic particles

1 Introduction

While in totally inclusive deep inelastic scattering (DIS) the quark chirality is conserved up to terms proportional to the quark masses, this is not the case in the Drell-Yan process. Here a quark-antiquark-pair is annihilated to a virtual photon, so that in the cross-section the quarks originating and ending in the same nucleon may carry different chirality. This gives raise to chiral odd distribution functions, which first appeared in the discussion of the transverse polarized Drell-Yan process [1]. This chiral odd distribution function is defined by a twist-2 operator and is called the transversity distribution h_1. Unlike the helicity asymmetry g_1, h_1 has no partonic interpretation in the chiral basis. Changing to the transversity basis [2] g_1 looses its partonic interpretation and h_1 gets one instead. It is interpreted as the probability to find a quark in a transversely polarized nucleon in an eigenstate of the transverse Pauli-Lubanski vector with eigenvalue $+1/2$, minus the same with eigenvalue $-1/2$.

Some experiments are planned to measure h_1 in the near future, especially at RHIC (BNL) and possibly by the COMPASS experiment at CERN — for a general review of the possibilities for measuring the transversity distribution see [3] —, so that there is need for theoretical predictions for h_1. The anomalous dimension which we will also consider in this contribution was calculated in [4]. In the nonrelativistic quark models g_1 and h_1 are identical. The positivity of parton probabilities implies the inequality $|h_1(x)| \leq f_1(x)$ and the Soffer inequality [5] $2\,|h_1(x)| \leq f_1(x) + g_1(x)$. A bag model calculation predict $|h_1(x)| \geq |g_1(x)|$, which may be correct in general [6]. For medium large Bjorken-x there exists a QCD-sum rule calculation [7], predicting a much smaller value for h_1 than the already mentioned bag model calculation.

We wish to contribute to this theoretical discussion by calculating the intrinsic uncertainty of the perturbative series due to IR-renormalon poles. We choose to discuss the product of an axialvector and a scalar current, which in first order of perturbation theory is generating the chiral-odd distribution function h_1. Thus the intrinsic uncertainty of the current product is a systematic error for the experimental measurement of h_1.

2 Definitions

2.1 Forward-scattering-amplitude

The definition of h_1 in terms of an operator matrix element reads [1,6,7]:

$$\begin{aligned} &\frac{i}{2}\int \frac{dy_+}{2\pi}\, e^{iy_+x}\big\langle p,s\big|\overline{\psi}(0)\sigma_{\mu\nu}\gamma_5\psi(y_+n)\big|p,s\big\rangle \\ &= h_1(x,q^2)\,(s_{\perp\mu}p_\nu - s_{\perp\nu}p_\mu) \\ &\quad + h_L(x,q^2)m^2\,(p_\mu n_\nu - p_\nu n_\mu)\,(s\cdot n) \\ &\quad + h_3(x,q^2)m^2\,(s_{\perp\mu}n_\nu - s_{\perp\nu}n_\mu) \end{aligned} \tag{1}$$

where $x = -q^2/(2p\cdot q)$ is the Bjorken variable, n^ν is a light cone vector with $n^2 = 0$ of dimension $(mass)^{-1}$, p is the proton momentum and s is the proton spin. $p^2 = m^2$, $s^2 = -1$ and $p \cdot s = 0$. $p \cdot n = 1$ and the transverse part of the spin vector is defined by the decomposition $s_\mu = (s\cdot n)p_\mu + (s\cdot p)n_\mu + s_{\perp\mu}$. The contraction of this expression with the light cone vector n^ν gives:

$$\begin{aligned} &\frac{i}{2}\int \frac{dy_+}{2\pi}\, e^{iy_+x}\big\langle p,s\big|\overline{\psi}(0)\sigma_{\mu\nu}n^\nu\gamma_5\psi(y_+n)\big|p,s\big\rangle \\ &= h_1(x,q^2)s_{\perp\mu} - h_L(x,q^2)m^2 n_\mu(s\cdot n). \end{aligned} \tag{2}$$

It is possible to relate the transversity distribution h_1 to the imaginary part of the forward scattering amplitude on the leading twist-level [7]:

$$T_\mu = \frac{i}{2}\int d^4y\, e^{iqy} \langle p, s| T\left(j_{\mu 5}(y) j_S(0) + j_S(y) j_{\mu 5}(0)\right) |p, s\rangle. \tag{3}$$

Here $j_{\mu 5}(y) = \overline{\psi}(y)\gamma_\mu\gamma_5\psi(y)$ is a axial-vector current, $j_S(y) = \overline{\psi}(y)\psi(y)$ is a scalar current and T denotes the time-ordered product. Summation over flavor indices is assumed. Equivalently one may use a time-ordered product of a vector-current $j_\mu(y) = \overline{\psi}(y)\gamma_\mu\psi(y)$ and a pseudoscalar current $j_5(y) = \overline{\psi}(y)\gamma_5\psi(y)$.

To prove the relation between this definition and the operator-definition (1) of the transversity distribution, one has to decompose the forward scattering amplitude into its Lorentz-structures using the conservation of either the axial-vector or the vector current. The conservation of the axial-vector current is correct only for the flavor nonsinglet current, so that the proof remains correct up to an order α_s-correction only. As the vector-current is conserved independently of the flavor combination under consideration we prefer the definition

$$T_\mu = \frac{i}{2}\int d^4y\, e^{iqy} \langle p, s| T\left(j_\mu(y) j_5(0) + j_5(y) j_\mu(0)\right) |p, s\rangle. \tag{4}$$

To find the realation to h_1 the current product has to be expanded collecting the terms with one incoming quark, one outgoing quark and one quark propagator $S(y) = i\,\partial\!\!\!/\,\Delta(y)$, where Δ is the Pauli-Jordan function. The Pauli-Jordan function is expanded on the light cone and only the leading term is taken into account. The imaginary part of the s-channel-term of the forward scattering amplitude takes the form:

$$\begin{aligned} \mathrm{Im}\, T_\mu =& \\ -\frac{1}{2\pi}&\int d^4y\, e^{iqy}\langle p, s| : \overline{\psi}(y)\sigma_{\mu\nu}y^\nu i\gamma_5\psi(0) : |p, s\rangle \delta'(y^2) \\ +\frac{1}{2\pi}&\int d^4y\, e^{iqy}\langle p, s| : \overline{\psi}(0)\sigma_{\mu\nu}y^\nu i\gamma_5\psi(y) : |p, s\rangle \delta'(y^2). \end{aligned} \tag{5}$$

Here $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ and $\delta'(y^2) = \left(\partial/\partial(y^2)\right)\delta(y^2)$. Substituting light cone variables $y_- = y_0 - y_3$ and $2y_+ = y_0 + y_3$, integrating by parts, using translation invariance, and putting this expression on the light cone by choosing y such that $y^2 = 0$, one gets

$$\begin{aligned} \mathrm{Im}\, T_\mu = -\frac{i}{2}\int \frac{dy_+}{2}\, e^{iy_+x} \\ \langle p, s| : \overline{\psi}(0)\sigma_{\mu\nu}n^\nu\gamma_5\psi(y_+n) : |p, s\rangle. \end{aligned} \tag{6}$$

This result has the form of (2), so that one gets a relation between the s-channel-part of the forward scattering amplitude and the transversity distribution by collecting the terms proportional to $s_{\perp\mu}$:

$$h_1(x)s_{\perp\mu} = \frac{1}{\pi}\, \mathrm{Im}\, T_\mu|_{s_{\perp\mu}}. \tag{7}$$

2.2 Light cone expansion and moments

The light cone expansion of the forward scattering amplitude may be writen as:

$$\begin{aligned} T_\mu|_{s_{\perp\mu}} = 2s_{\perp\mu}\sum_{m=0}^{\infty} C_m\left(\frac{Q^2}{\mu^2}, \alpha_s\right) A_m(\mu^2)\,\omega^{m+1} \\ +\text{higher twist} \end{aligned} \tag{8}$$

where $\omega = 1/x$. A_m are the reduced matrix elements of the local twist-2 operators relevant for h_1:

$$\begin{aligned} \langle p, s|\overline{\psi}\sigma^{\lambda\{\rho} i\gamma_5 iD^{\mu_1}\cdots iD^{\mu_m\}}\psi|p, s\rangle = \\ 2A_m(s^\lambda p^{\{\rho} - p^\lambda s^{\{\rho})p^{\mu_1}\cdots p^{\mu_m\}}\psi - \text{traces}. \end{aligned} \tag{9}$$

The brackets denote total symmetrization of all included indices. The symmetrization and the subtraction of the traces are necessary to extract the leading twist part of the matrix element. The moments of h_1 have to be defined as $M_m = C_m A_m$ to get the general expression:

$$2s_{\perp\mu}M_n = \frac{1}{\pi}\int_0^1 dx\, x^n \left(Im\, T_\mu(x) + (-)^n Im\, T_\mu(-x)\right)|_{s_{\perp\mu}}. \tag{10}$$

Using (7) the moments become:

$$M_n = \frac{1}{2}\int_0^1 dx\, x^n \left(h_1(x) + (-)^n h_1(-x)\right) \tag{11}$$

for $n = 0, 1, \ldots$ This result coincides with the expression found by Jaffe and Ji [6].

The operators in (9) have a well defined behaviour under charge conjugation: they are C-odd for even n and C-even for odd n. This change in sign corresponds to the relative sign of $h_1(x)$ and $h_1(-x)$ in the moments (11). To obtain the correct relation between the antiquark and the quark transversity distribution one may look at the first moment:

$$M_0 = \frac{1}{2}\int_0^1 dx\, \left(h_1(x) + h_1(-x)\right). \tag{12}$$

As this expression is odd under charge conjugation, the antiquark transversity distribution should be $\overline{h}_1(x) = -h_1(-x)$. It follows that the contributions of sea quarks cancel for even moments in general and only the valence quarks contribute, while the sea quark contributions add for odd moments.

3 Renomalon ambiguity of $j_{\mu 5} j_S$

We are calculating the IR-renormalon contribution [8] to the structure function defined by (3) and (7). As its twist-2 part is related to the transversity distribution h_1, the IR-renormalon contribution may be interpreted as intrinsic ambiguity of the perturbative expansion to h_1. This interpretation is justified by the fact, that the calculation of the renormalon ambiguity involves only one loop corrections using a resummed coupling. The correspondence of the current product and the nonsinglet distribution function h_1 is exact up to first order in α_s, so that this correspondence applies for the ambiguities in a one loop calculation also. Nevertheless one should be aware, that we calculate the renormalon uncertainty of the current product and not of h_1, so that we give only an estimate for the intrinsic uncertainty of h_1.

Perturbative series in QCD are asymptotic ones, implying that the radiative corrections of higher and higher orders get smaller only up to a finite order m_0 and diverge for larger orders in α_s. The uncertainty of the series is of the order of the smallest contribution and has the form of a power correction [9], so that the QCD perturbative series on the lowest twist level may be written as ($a_s = \alpha_s/(4\pi)$):

$$C_n\left(Q^2=\mu^2, a_s\right) = \sum_{k=0}^{m_0} B_n^{(k)} a_s^k + C_n^{(1)} \frac{\Lambda_C^2 e^{-C}}{Q^2} + C_n^{(2)} \frac{\Lambda_C^4 e^{-2C}}{Q^4} + \mathcal{O}\left(\frac{1}{Q^6}\right). \quad (13)$$

Note that the $1/Q$ term is not present in the above expression. It was shown for Drell-Yan on the one gluon exchange level that these corrections are cancelled by higher order perturbative contributions [10] and that the $1/Q^2$-term is the leading power correction. The uncertainty may be determined by the exact calculation of the perturbative corrections up to the order $m_0(Q^2)$, which is a very demanding procedure.

Instead we calculate the forward scattering amplitude (3) on the one gluon exchange level, using a Borel transformed effective gluon propagator [11]:

$$\mathcal{B}_{1/a_s}[a_s D_{\mu\nu}^{ab}(k)](u) = \delta^{ab} \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2} \left(\frac{\mu^2 e^{-C}}{-k^2}\right)^{\beta_0 u}. \quad (14)$$

C corrects for the renormalization scheme dependence ($C = -\frac{5}{3}$ for $\overline{MS}$-scheme), μ is the renormalization scale, and u is the Borel parameter. The effective gluon propagator is constructed by replacing the coupling a_s by the running coupling constant, that is by a resummation of all quark- and gluon-loop insertions in one gluon-propagator. In the first order of a_s this propagator leads to exact results. Looking at higher order corrections the restriction on one exchanged effective gluon corresponds to the large N_f-limit [12], where N_f denotes the number of quark flavors. The next-to-leading N_f-terms are approximated by naive-nonabelianization (NNA) [13]. This corresponds to the replacement of the one loop QED-beta-function by the QCD-beta-function $\beta_0 = 11 - \frac{2}{3}N_f$ or equivalently to $N_f \rightarrow N_f - \frac{33}{2}$. The quality of this approximation has been checked [14,15] for the unpolarized structure functions F_2 and F_L and for the polarized structure function g_1 by comparing the NNA perturbative coefficients with the known exact ones. This comparison gave very reasonable results for F_L and g_1, while the results for F_2 are less convincing.

Formally, asymptotic freedom is destroyed in the large N_f-limit. One should recognize that the large N_f-limit is used to select graphs and has to be understood as a definition of an approximation procedure. At the end N_f will be set to 4, so that β_0 stays in an asymptotic free region. This procedure is technically analogous to the use of the large N_c-limit [16], even if the physical content is different.

In the Borel plane the ambiguity of the truncated perturbative series in (13) is reflected in IR-renormalon poles, which hinder an unambigous inverse Borel transformation. This ambiguity of the perturbative series may be interpreted as twist-4 contribution of the structure function defined by (3) and (7). Such estimations of higher twist corrections gave very reasonable results [17,18,14, 15]. However the transversity distribution is defined by pure twist-2 operators. This means that the renormalon ambiguity we are calculating is not used to give an estimate of a twist-4 part but is an intrinsic ambiguity of the whole twist-2 perturbative series for h_1.

So let us calculate the Borel-transformed s-channel forward scattering amplitude T_μ in (3) on the one gluon exchange level using the effective gluon propagator (14). The result is a series in $\omega = 1/x$ which has to be compared with (8)

$$\mathcal{B}_{\frac{1}{a_s}}\left(T_\mu\right)\Big|_{s_{\perp\mu}} = s_{\perp\mu} \sum_{m=0}^{\infty} \mathcal{B}_{\frac{1}{a_s}}\left(C_m\left(\frac{Q^2}{\mu^2}, a_s\right)\right) \times A_m(\mu^2)\, \omega^{m+1} + \text{higher twist} \quad (15)$$

where the reduced matrix element was determined at the tree level to be $A_n = 1$ and a factor 2 is missing because the exchange graphs are not included in this expression due to the restriction on the s-channel contribution. We obtain for the Borel transformed Wilson coefficient:

$$\begin{aligned}
\mathcal{B}_{\frac{1}{a_s}}\left[C_n\left(\frac{Q^2}{\mu^2}, a_s\right)\right](s) \;=\; & C_F\left(\frac{\mu^2 e^{-C}}{Q^2}\right)^s \\
\Bigg\{ & \frac{1}{s}\left[\frac{5}{1+s} - \frac{\Gamma(1+s+n)}{\Gamma(1+s)\Gamma(1+n)} + \sum_{k=1}^{n} \frac{\Gamma(s+k)}{\Gamma(1+s)\Gamma(1+k)} \frac{4k}{1+s+k}\right] \\
& + \frac{1}{1-s}\left[\frac{4}{1+s} - \frac{2\Gamma(1+s+n)}{\Gamma(1+s)\Gamma(1+n)} + \sum_{k=1}^{n} \frac{\Gamma(s+k)}{\Gamma(1+s)\Gamma(1+k)} \frac{4(1+k)}{1+s+k}\right] \\
& + \frac{1}{2-s}\left[\frac{1}{1+s} + \frac{\Gamma(1+s+n)}{\Gamma(1+s)\Gamma(1+n)} + \sum_{k=1}^{n} \frac{\Gamma(s+k)}{\Gamma(1+s)\Gamma(1+k)} \frac{2}{1+s+k}\right] \Bigg\}
\end{aligned} \quad (16)$$

where $s = \beta_0 u$ replaces the Borel parameter u. We find IR-renormalon poles at $s = 0, 1, 2$, as usual for DIS.

This is not a general statement as in the case of e^+e^--fragmentation one obtains an infinite sum of poles with even powers of $1/Q$ [17]. The Wilson coefficient has still to be renormalized.

In the Borel-plane the ambiguity $C_n^{(k)}$ of the perturbative series in (13) can be rediscovered as ambiguity of the inverse Borel transformation due to the IR-renormalon poles or in other words as the imaginary part of the Laplace integral:

$$\sum_k C_n^{(k)} \left(\frac{\Lambda_C^2 e^{-C}}{Q^2}\right)^k = \frac{1}{\pi\beta_0} \mathrm{Im} \int_0^\infty ds\, e^{-s/(\beta_0 a_s)} \times \mathcal{B}_{\frac{1}{a_s}} \left[C_n \left(\frac{Q^2}{\mu^2}, a_s\right)\right](s). \quad (17)$$

We get

$$C_n^{(1)} = \pm \frac{2C_F}{\beta_0} \left(3 + n - \frac{2}{1+n} - \frac{2}{2+n} - 2\sum_{k=1}^{n} \frac{1}{k}\right)$$
$$C_n^{(2)} = \pm \frac{C_F}{\beta_0} \left(3 + \frac{1}{2}(1+n)(4+n) - \frac{2}{1+n} - \frac{2}{2+n} - \frac{2}{3+n} - 2\sum_{k=1}^{n} \frac{1}{k}\right). \quad (18)$$

The signs of these twist-2 uncertainty terms remain undetermined, because it is not clear in which way the pole should be circumvented in the Laplace integral.

In the following the ambiguity of the Laplace integral is interpreted as an intrinsic uncertainty of the twist-2 transversity distribution. The ratio of the moments of this uncertainty h_1^{IR} and the moments of h_1 is expanded up to the order a_s/Q^2

$$M_n^{\mathrm{IR}} = \frac{C_n^{(1)} \frac{\Lambda^2 e^{-C}}{Q^2} + \mathcal{O}\left(\frac{a_s}{Q^2}, \frac{1}{Q^4}\right)}{\sum_{k=0}^{m_0} B_n^{(k)} a_s^k + C_n^{(1)} \frac{\Lambda^2 e^{-C}}{Q^2} + \mathcal{O}\left(\frac{a_s}{Q^2}, \frac{1}{Q^4}\right)} M_n \approx \left[\frac{C_n^{(1)}}{B_n^{(0)}} \frac{\Lambda^2 e^{-C}}{Q^2} + \mathcal{O}\left(\frac{a_s}{Q^2}, \frac{1}{Q^4}\right)\right] M_n. \quad (19)$$

Here the truncated perturbative series (13) was used for the moments of h_1. The lowest order twist-2 perturbative coefficient $B_n^{(0)}$ is determined by the tree-graph and is 1. As an illustration we will insert for M_n a theoretical model prediction for the transversity distribution.

From (18) and (19) we can calculate the IR-renormalon uncertainty U_n for each moment, where

$$\int_0^1 dx\, x^n \left(h_1(x) - (-)^n \overline{h}_1(x)\right) = A_n^{(0)} (1 \pm U_n) + \mathcal{O}(a_s), \quad (20)$$

where $A_n^{(0)}$ denotes the leading contribution to the moments. At $Q^2 = 4\,\mathrm{GeV}^2$ and with $\Lambda_{\overline{MS}} = 200\,\mathrm{MeV}$, $N_f = 4$, $C_F = \frac{4}{3}$ and $\beta_0 = 11 - \frac{2}{3}N_f$ we find:

$$\int_0^1 dx\, \left(h_1(x) - \overline{h}_1(x)\right) = A_0^{(0)} + \mathcal{O}(a_s),$$

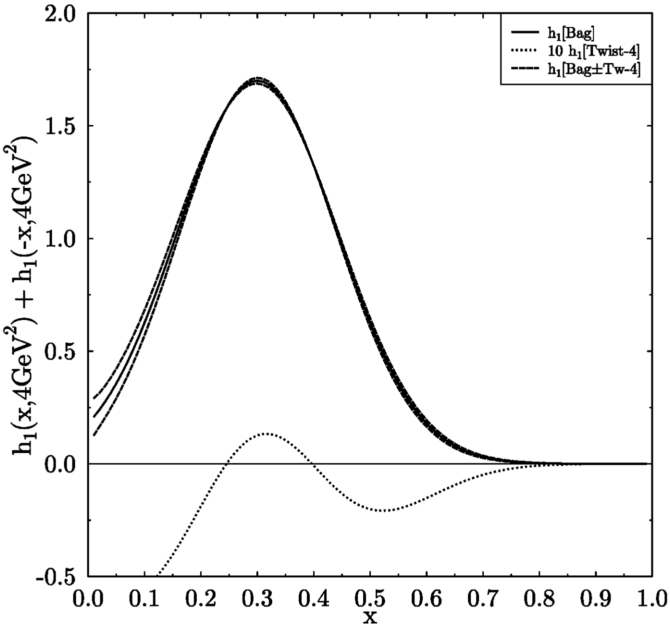

Fig. 1. The bag model calculation [6] for $h_1 - \overline{h}_1$ (full line) and the IR-renormalon ambiguity evaluated at $Q^2 = 4\,\mathrm{GeV}^2$ multiplied by a factor of 10 (dotted line). There are two changes in sign at $x \approx 0.25$ and $x \approx 0.4$. The ambiguity was added and subtracted from the bag model calculation (dashed lines), ($\Lambda_{\overline{MS}} = 200$ MeV, and $N_f = 4$)

$$\int_0^1 dx\, x \left(h_1(x) + \overline{h}_1(x)\right) = A_1^{(0)}(1 \pm 0.0057) + \mathcal{O}(a_s),$$
$$\int_0^1 dx\, x^2 \left(h_1(x) - \overline{h}_1(x)\right) = A_2^{(0)}(1 \pm 0.014) + \mathcal{O}(a_s),$$
$$\int_0^1 dx\, x^3 \left(h_1(x) + \overline{h}_1(x)\right) = A_3^{(0)}(1 \pm 0.024) + \mathcal{O}(a_s),$$
$$\int_0^1 dx\, x^4 \left(h_1(x) - \overline{h}_1(x)\right) = A_4^{(0)}(1 \pm 0.036) + \mathcal{O}(a_s). \quad (21)$$

We get no IR-renormalon uncertainty for the first moment. The IR-renormalon uncertainty becomes larger for higher moments, so that we expect larger ambiguities of the transversity distribution in the region of larger Bjorken-x. For higher moments the contribution of $\overline{h}_1$ can be considered as marginal, so that the above ambiguities should remain approximately correct for $\int_0^1 dx x^n h_1(x)$ with $n > 2$. This is of course not the case for the first and the second moment ($n = 1$). A rough estimate gives rise to a sea-quark effect of the same order as the calculated uncertainty.

From the moments in (19) the valence quark transversity distribution $h_1 - \overline{h}_1$ may be reconstructed as a function of Bjorken-x. The result is a convolution integral:

$$h_1^{IR}(x, Q^2) - \overline{h}_1^{IR}(x, Q^2) = \frac{\Lambda^2 e^{-C}}{Q^2} \int_x^1 \frac{dy}{y} \widetilde{C}^{(1)}(y) \times \left\{ h_1\left(\frac{x}{y}\right) - \overline{h}_1\left(\frac{x}{y}\right)\right\} \quad (22)$$

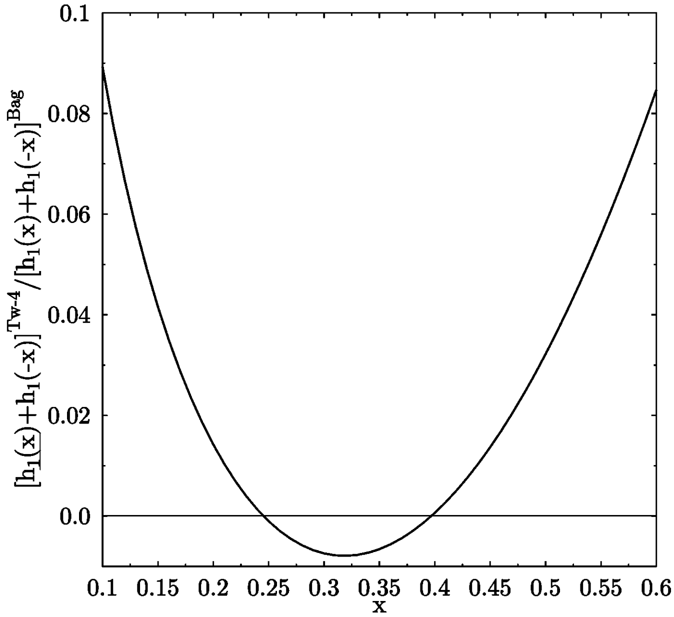

Fig. 2. The relative magnitude of the IR-renormalon ambiguity with respect to the bag model calculation [6] for $h_1 - \overline{h}_1$ ($\Lambda_{\overline{MS}} = 200$ MeV, $Q^2 = 4\,\mathrm{GeV}^2$ and $N_f = 4$)

where $\widetilde{C}^{(k)}(x)$ is defined by

$$C_n^{(k)} \;=\; \int_0^1 dx\, x^n \widetilde{C}^{(k)}(x) \tag{23}$$

and we find for the two first IR-renormalon ambiguities:

$$\begin{aligned}\widetilde{C}^{(1)}(x) &= \pm\frac{2C_F}{\beta_0}\Bigg\{\frac{2}{(1-x)_+} + 3\delta(x-1) \\ &\quad -\delta'(x-1) - 2x - 2\Bigg\} \\ \widetilde{C}^{(2)}(x) &= \pm\frac{2C_F}{\beta_0}\Bigg\{\frac{1}{(1-x)_+} + \frac{5}{2}\delta(x-1) \\ &\quad -\delta'(x-1) + \frac{x}{4}\delta''(x-1) - x^2 - x - 1\Bigg\}.\end{aligned} \tag{24}$$

The IR-renormalon ambiguities shown in Fig. 1 are calculated using the bag model calculation of [6] for $h_1(x)$. The large N_f-limit is most reliable in the region of medium large Bjorken-x. For small x the neglection of multiple gluon exchange is no longer justified. On the other hand the influence of the hadronic spectrum makes a pure perturbative calculation insufficient for large x. In the region of best accuracy ($0.2 < x < 0.45$) the uncertainty does not become bigger than 1% (see Fig. 2). One can expect a sizeable IR-renormalon uncertainty of up to 10% for $0.5 < x < 0.6$.

4 Conclusions

We gave an estimate of the IR-renormalon ambiguity for the valence quark transversity distribution. The ambiguity is smaller than 1% in the region of best accuracy of the NNA-approximation. For $x \approx 0.6$ this systematic error becomes important reaching about 10%. Thus the renormalon uncertainty should be taken into accout when interpreting measurements of h_1 in this region of x.

We thank L. Szymanowski and S. Hofmann for helpful discussions. This work has been supported by BMBF. A.S. thanks also the MPI für Kernphysik in Heidelberg for support.

References

1. J. Ralston, Davison E. Soper, Nucl. Phys. **B152** (1979), 109. R.D. Tangerman, P.J. Mulders, Phys. Rev. **D51** (1995), 3357-3372
2. G.R. Goldstein, Ann. Phys. **98** (1976), 128; Ann. Phys. **142** (1982), 219; Ann. Phys. **195** (1989), 213
3. R.L. Jaffe, The context of high energy QCD spin physics, in: Spin Structure of the nucleon. Wako, Japan 1995, S. 2-22
4. X. Artru, M. Mekhfi, Z.Phys. **C45** (1990), 669-676
5. J. Soffer, Phys. Rev. Lett. **74** (1995), 1292; G.R. Goldstein, R.L. Jaffe, Xiangdong Ji, Phys. Rev. **D52** (1995), 5006
6. R.L. Jaffe, Xiangdong Ji, Phys. Rev. Lett. **67** (1991), 552-555; Nucl. Phys. **B375** (1992), 527
7. B.L. Ioffe, A. Khodjamirian, Phys. Rev. **D51** (1995), 3373-3380
8. A.H. Mueller, Nucl. Phys. **B250** (1985), 327-350; V.I. Zakharov, Nucl. Phys. **B385** (1992), 452; A.H. Mueller, The QCD perturbation series, in: P.M. Zerwas, H.A. Kastrup (Hrsg.), QCD - 20 years later, Aachen 6/1992. World Scientific, Singapore, New Jersey, London, Hong Kong 1993, S. 162-171; P. Ball, M. Beneke, V.M. Braun, Nucl. Phys. **B452** (1995), 563
9. A.H. Mueller, Phys. Lett. **B308** (1993), 355; V.M. Braun, QCD renormalons and higher twist effects, in: Proceeding of 30th Rencontres de Moriond 1995: QCD and High Energy Hadronic Interaction, Meribel les Allues 1995
10. M. Beneke, V.M. Braun, Nucl. Phys. **B454** (1995), 253
11. M. Beneke, Nucl. Phys. **B405** (1993), 424-450; M. Beneke, V.M. Braun, Nucl. Phys. **B426** (1994), 301-343
12. A.N. Vasilev, Yu.M. Pismak, J.R. Honkonen, Theor. Math. Phys. **46** (1981), 157; Theor. Math. Phys. **47** (1981), 291; J.A. Gracey, Mod. Phys. Lett. **A7** (1992), 1945; Int. J. Mod. Phys. **A8** (1993), 2465; Phys. Lett. **B317** (1993), 415; Phys. Lett. **B318** (1993), 177; AIHENP (1995), 259-264; Phys. Lett. **B373** (1996), 178-184; M. Neubert, Phys. Rev. **D51** (1995), 5924
13. D.J. Broadhurst, A.G. Grozin, Phys. Rev. **D52** (1995), 4082; M. Beneke, V.M. Braun, Phys. Lett. **B348** (1995), 513-520
14. E. Stein, M. Meyer-Hermann, L. Mankiewicz, A. Schäfer, Phys. Lett. **B376** (1996), 177-185
15. M. Meyer-Hermann, M. Maul, L. Mankiewicz, E. Stein, A. Schäfer, Phys. Lett. **B383** (1996), 463-469; Phys. Lett. **B393** (E) (1997), 487
16. G. t'Hooft, Nucl. Phys. **B72** (1974), 461; Edward Witten, Nucl. Phys. **B160** (1979), 57
17. M. Dasgupta, B.R. Webber, Phys. Lett. **B382** (1996), 273-281
18. Yu.L. Dokshitser, G. Marchesini, B.R. Webber, Nucl. Phys. **B469** (1996), 93-142